

Ex.15 设 A, B, C 是同阶的方阵且

$$B = E + AB, \quad C = A + CA.$$

证明. $B - C = E$.

证明. 根据条件,

$$B = E + AB \implies B - AB = E \implies (E - A)B = E \implies B^{-1} = E - A.$$

$$C = A + CA \implies C - CA = A \implies C(E - A) = A \implies CB^{-1} = A.$$

所以, $C = AB$. 于是,

$$B = E + AB \implies B = E + C \implies B - C = E.$$